

Grade 8 Mathematics Item Specification C1 TB

Claim 1: Concepts and Procedures

Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: Expressions and Equations

Target B [m]: Work with radicals and integer exponents. (DOK Level 1)

Tasks for this target will require students to select or produce equivalent numerical expressions based on properties of integer exponents.

Other tasks will ask students to solve simple equations whose solutions can be expressed as square and cube roots, often expressing their answers approximately, using one of the approximations from 8.NS Target A.

Other tasks will ask students to represent very large and very small quantities in scientific notation, and perform operations on numbers written in scientific notation.

<p>Standards: 8.EE.A, 8.EE.A.1, 8.EE.A.2, 8.EE.A.3, 8.EE.A.4</p>	<p>8.EE.A Work with radicals and integer exponents.</p> <p>8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. <i>For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.</i></p> <p>8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.</p> <p>8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. <i>For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.</i></p> <p>8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.</p>
<p>Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling:</p> <p>7.EE.A, 7.EE.A.1, 7.EE.A.2, 7.EE.B, 7.EE.B.3</p> <p>A-SSE.A, A-SSE.A.1, A-SSE.A.2, A-SSE.B, A-SSE.B.3</p>	<p>Related Grade 7 Standards</p> <p>7.EE.A Use properties of operations to generate equivalent expressions.</p> <p>7.EE.A.1 Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.</p> <p>7.EE.A.2 Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. <i>For example, $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05.”</i></p> <p>7.EE.B Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</p> <p>7.EE.B.3 Solve multi-step, real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the</p>

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	<p>reasonableness of answers using mental computation and estimation strategies. <i>For example: If a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50. If you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.</i></p> <p>Related High School Standards</p> <p>A-SSE.A Interpret the structure of expressions</p> <p>A-SSE.A.1 Interpret expressions that represent a quantity in terms of its context.*</p> <ol style="list-style-type: none"> Interpret parts of an expression, such as terms, factors, and coefficients. Interpret complicated expressions by viewing one or more of their parts as a single entity. <i>For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.</i> <p>A-SSE.A.2 Use the structure of an expression to identify ways to rewrite it. <i>For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</i></p> <p>A-SSE.B Write expressions in equivalent forms to solve problems.</p> <p>A-SSE.B.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.</p> <ol style="list-style-type: none"> Factor a quadratic expression to reveal the zeros of the function it defines. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. Use the properties of exponents to transform expressions for exponential functions. <i>For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.</i>
DOK Levels:	1, 2
Achievement Level Descriptors:	
<p>RANGE</p> <p>Achievement Level Descriptor (Range ALD)</p> <p>Target B: Work with radicals and integer exponents.</p>	<p>Level 1 Students should be able to identify and calculate square roots of familiar perfect squares and calculate the square of integers. They should be able to translate between standard form and scientific notation.</p> <p>Level 2 Students should be able to identify and calculate the cube root of familiar perfect cubes and calculate the cube of integers. They should be able to use appropriate tools (e.g., calculator, pencil and paper) to translate large or small numbers from scientific to standard notation. They should be able to work with and apply the properties of integer exponents of degree 2 or less in order to produce or identify equivalent numerical expressions.</p> <p>Level 3 Students should be able to identify that the square root of 2 is irrational, calculate or approximate to an appropriate degree of precision the square or cube of a rational number, solve quadratic and cubic monomial equations, and represent the solution as a square or cube root, respectively. They should be able to work with and perform operations with scientific notation and work with and</p>

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	<p>apply the properties of integer exponents in order to produce or identify equivalent numerical expressions.</p> <p>Level 4 Students should be able to use scientific notation and choose units of appropriate size for realistic measurements, solve binomial quadratic and cubic equations, and represent the solution as a square or cube root, respectively.</p>
Evidence Required:	<ol style="list-style-type: none"> 1. The student generates equivalent numerical expressions by applying the properties of integer exponents. 2. The student represents solutions to equations of the form $x^2 = p$ using square root symbols. 3. The student represents solutions to equations of the form $x^3 = p$ using cube root symbols. 4. The student states how many times as large or as small one number, written as a single digit times a power of 10, is than another, to estimate very large or very small quantities. 5. The student performs operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.
Allowable Response Types:	Multiple Choice, single correct response, Multiple Choice, multiple correct response, Equation/Numeric
Allowable Stimulus Materials:	numbers written as a single digit multiplied by an integer power of 10, numbers in scientific notation
Construct-Relevant Vocabulary:	rational number, irrational number, scientific notation, decimal notation, exponent, power, base, radical, square root, cube root, perfect square, perfect cube, exponent
Allowable Tools:	None
Target-Specific Attributes	<p>Exponents should be integers.</p> <p>Roots should be square roots or cube roots of positive rational numbers.</p> <p>Perfect squares should be positive rational numbers that are less than or equal to 400.</p> <p>Perfect cubes should be positive rational numbers that are less than or equal to 1000.</p>
Non-Targeted Constructs:	
Accessibility Guidance:	<p>Item writers should consider the following Language and Visual Element/Design guidelines¹ when developing items.</p> <p>Language Key Considerations:</p> <ul style="list-style-type: none"> • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual

¹ For more information, refer to the General Accessibility Guidelines at:

<http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf>

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	<p>names and references</p> <p>Visual Elements/Design Key Considerations:</p> <ul style="list-style-type: none"> • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics <p>Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.²</p>
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² For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

<p>Task Model 1</p> <p>Response Type: Multiple Choice, multiple correct response</p> <p>DOK Level 1</p> <p>8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. <i>For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.</i></p> <p>Evidence Required: 1. The student generates equivalent numerical expressions by applying the properties of integer exponents.</p> <p>Tools: None</p>	<p>Prompt Features: The student determines equivalent numerical expressions by applying the properties of integer exponents.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> • The bases are integers between –5 and 5. • Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> ○ One-step multiplication or division expression. ○ Multi-step multiplication and division expression. ○ All positive integer exponents. ○ Combination of negative and positive integer exponents. ○ Combination of negative and positive integer bases. ○ Including a base to the 1st power with no exponent given. <p>TM1a Stimulus: The student is presented with a numerical expression involving integer exponents.</p> <p>Example Stem: Select all expressions equivalent to $(4^5 \cdot 4^{-3})^{-2}$.</p> <p>A. $\frac{1}{256}$ B. 256 C. $4^{-10} \cdot 4^6$ D. $4^3 \cdot 4^{-5}$</p> <p>Answer Choices: There should be four answer choices with at least two correct choices that will be expressions or numbers. Incorrect answer choices will reflect errors in understanding negative exponents or error applying exponent rules.</p> <p>Rubric: (1 point) Student selects all equivalent expressions (e.g., A and C)</p> <p>Response Type: Multiple Choice, multiple correct response</p>
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<p>Task Model 1</p> <p>Response Type: Equation/Numeric</p> <p>DOK Level 1</p> <p>8.EE.A.1 Know and apply the properties of integer exponents to generate equivalent numerical expressions. <i>For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.</i></p> <p>Evidence Required: 1. The student generates equivalent numerical expressions by applying the properties of integer exponents.</p> <p>Tools: None</p>	<p>Prompt Features: The student determines a value that makes two expressions equivalent by applying the properties of integer exponents.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> • The missing value may represent any exponent. • The bases are integers between –5 and 5. • Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> ○ One-step multiplication or division expression. ○ Multi-step multiplication and division expression. ○ All positive integer exponents. ○ Combination of negative and positive integer exponents. ○ Combination of negative and positive integer bases, including a base to the 1st power with no exponent given. <p>TM1b</p> <p>Stimulus: The student is presented with an equation with numerical expressions involving integer exponents.</p> <p>Example Stem: Enter the value of n that makes the equation $4^5 \cdot 4^n = 4^{15}$ true.</p> <p>Rubric: (1 point) Student applies properties of exponents correctly and enters the correct response (e.g., 10).</p> <p>Response Type: Equation/Numeric</p>
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<p>Task Model 2</p> <p>Response Type: Multiple Choice, multiple correct response</p> <p>DOK Level 1</p> <p>8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.</p> <p>Evidence Required: 2. The student generates solutions to equations of the form $x^2 = p$ using square root symbols.</p> <p>Tools: None</p>	<p>Prompt Features: The student generates solutions to an equation of the form $x^2 = p$ using square root symbols.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> • Square roots of non-perfect squares must be in radical form as answer choices • Four to six answer choices may be given, including positive and negative values. • Item difficulty can be adjusted via these methods: <ul style="list-style-type: none"> ◦ p is a perfect square. ◦ x is a positive integer or rational number. ◦ x is a negative integer or rational number. ◦ p is not a perfect square. <p>TM2</p> <p>Stimulus: The student is presented with an equation in the form $x^2 = p$, where p is a positive rational number.</p> <p>Example Stem 1: Select all possible values for x that solve the equation $x^2 = 200$.</p> <p>A. $10\sqrt{20}$ B. $100\sqrt{2}$ C. $10\sqrt{2}$ D. $\sqrt{200}$</p> <p>Example Stem 2: Select all possible values for x in the equation $x^2 = 200$.</p> <p>A. $10\sqrt{2}$ B. $10\sqrt{20}$ C. $20\sqrt{10}$ D. $-10\sqrt{2}$ E. $-10\sqrt{20}$ F. $-20\sqrt{10}$</p> <p>Answer Choices: At least two answer choices must be correct responses. Incorrect answer choices should reflect errors in inappropriately factoring and not taking the square root of a factor.</p> <p>Rubric: (1 point) Student selects all the correct expressions (e.g., C and D; A and D).</p> <p>Response Type: Multiple Choice, multiple correct response</p>
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<p>Task Model 3</p> <p>Response Type: Multiple Choice, multiple correct response</p> <p>DOK Level 1</p> <p>8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.</p> <p>Evidence Required: 3. The student generates solutions to equations of the form $x^3 = p$ using cube root symbols.</p> <p>Tools: None</p>	<p>Prompt Features: The student generates solutions to equations of the form $x^3 = p$ using cube root symbols.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> • Cube roots of non-perfect cubes must be in radical form as answer choices. • Four to six answer choices may be given, including positive and negative values. • Item difficulty can be adjusted via these methods: <ul style="list-style-type: none"> ◦ p is a perfect cube. ◦ x is a positive integer or rational number. ◦ x is a negative integer or rational number. ◦ p is not a perfect cube. <p>TM3</p> <p>Stimulus The student is presented with an equation of the form $x^3 = p$, where p is a positive rational number.</p> <p>Example Stem: Select all possible values for x in the equation, $x^3 = 250$.</p> <p>A. $5\sqrt[3]{2}$ B. $\sqrt[3]{250}$ C. $5\sqrt[3]{10}$ D. $25\sqrt[3]{10}$</p> <p>Answer Choices: At least two answer choices must be correct responses. Incorrect answer choices should reflect errors in inappropriately factoring and taking the square root instead of cube root.</p> <p>Rubric: (1 point) Student selects all the correct expressions (e.g., A and B).</p> <p>Response Type: Multiple Choice, multiple correct response</p>
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<p>Task Model 4</p> <p>Response Type: Multiple Choice, single correct response</p> <p>DOK Level 1</p> <p>8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities and to express how many times as much one is than the other. <i>For example, estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.</i></p> <p>Evidence Required: 4. The student states how many times as large or as small one number, written as a single digit times a power of 10, is than another to estimate very large or very small quantities.</p> <p>Tools: None</p> <p>Version 3 Update: Retired example stem 2 from TM4.</p>	<p>Prompt Features: The student identifies approximately how many times as large or as small one number is than another.</p> <p>Stimulus Guidelines: Item difficulty can be adjusted via these methods:</p> <ul style="list-style-type: none"> Two expressions in the form $a \times 10^b$ with a as the same single digit whole number in both expressions and b is positive in both expressions Two expressions in the form $a \times 10^b$ with a as the same single digit whole number in both expressions and b is negative in both expressions Two expressions in the form $a \times 10^b$ with a as the same single digit whole number in both expressions; b is negative in one expression and b is positive in the other expression OR two expressions with a as a different single digit whole number in both expressions and b is negative in both expressions or b is positive in both expressions Two expressions in the form $a \times 10^b$ with a as a different single digit whole number in both expressions; b is negative in one expression and b is positive in the other expression <p>TM4 Stimulus: The student is presented with quantities that should be expressed as a single digit times a power of ten.</p> <p>Example Stem: How many times larger than 2×10^3 is 6×10^6?</p> <p>A. 3×10^2 B. 3×10^3 C. 6×10^6 D. 12×10^9</p> <p>Response Type: Multiple Choice, single correct response</p> <p>Rubric: (1 point) Student selects the correct expression (e.g., B).</p> <p>Response Type: Multiple Choice, single correct response</p>
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<p>Task Model 5</p> <p>Response Type: Multiple Choice, single correct response</p> <p>DOK Level 1</p> <p>8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.</p> <p>Evidence Required: 5. The student performs operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.</p> <p>Tools: None</p>	<p>Prompt Features: The student is prompted to perform operations on numbers expressed in scientific notation or decimal form.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none"> Numbers may be in scientific notation or decimal form. Context should be familiar to 13–15 year olds. Items include two expressions in the form of $a \times 10^b$ that are either being added, subtracted, multiplied, or divided. Item difficulty can be adjusted via these methods: <ul style="list-style-type: none"> The exponent, b, for the two expressions could be either both positive, both negative, or a combination. The variable, a, for the two expressions could be both whole numbers, both decimals, or a combination. <p>TM5 Stimulus: The student is presented with very large or very small quantities with or without context.</p> <p>Example Stem 1: Approximately 7.5×10^5 gallons of water flow over a waterfall each second. There are 8.6×10^4 seconds in 1 day.</p> <p>Enter the approximate number of gallons of water that flow over the waterfall in 1 day.</p> <p>A. 6.45×10^{21} B. 6.45×10^{20} C. 6.45×10^{10} D. 6.45×10^9</p> <p>Answer Choices: Answer choices are numbers in scientific notation.</p> <p>Rubric: (1 point) Student gives the correct value. (e.g., C).</p> <p>Response Type: Multiple Choice, single correct response</p> <p>Example Stem 2: Which value is closest to $(6 \times 10^6) + (2 \times 10^4)$?</p> <p>A. 8.0×10^{10} B. 8.0×10^6 C. 6.0×10^{10} D. 6.0×10^6</p> <p>Rubric: (1 point) Student enters the correct value (e.g., D).</p> <p>Response Type: Multiple Choice, single correct response</p>
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