

<p>Claim 1: Concepts and Procedures Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.</p>	
<p>Content Domain: Statistics and Probability</p>	
<p>Target J [s]: Investigate patterns of association in bivariate data. (DOK Levels 1, 2)</p> <p>Tasks for this target will often be paired with 8.F Target F and ask students to determine the rate of change and initial value of a line suggested by examining bivariate data. Interpretations related to clustering, outliers, positive or negative association, linear and nonlinear association will primarily be presented in context by pairing this target with those from Claims 2 and 4.</p>	
<p>Standards: 8.SP.A, 8.SP.A.1, 8.SP.A.2, 8.SP.A.3, 8.SP.A.4</p>	<p>8.SP.A Investigate patterns of association in bivariate data</p> <p>8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.</p> <p>8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.</p> <p>8.SP.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. <i>For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.</i></p> <p>8.SP.A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. <i>For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?</i></p>
<p>Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling:</p> <p>7.SP.A, 7.SP.A.1, 7.SP.A.2, F-IF-B, F-IF.B.4, F-LE.B.5, S-ID.B, S-ID.B.5, S-ID.B.6</p>	<p>Related Grade 7 standards</p> <p>7.SP.A Use random sampling to draw inferences about a population</p> <p>7.SP.A.1 Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.</p> <p>7.SP.A.2 Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.</p>

	<p>Related High School Standards</p> <p>F-IF.B Interpret functions that arise in application in terms of the context. F-IF.B.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i> F-LE.B.5 Interpret the parameters in a linear or exponential function in terms of a context. S-ID.B Summarize, represent, and interpret data on two categorical and quantitative variables S-ID.B.5 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. S-ID.B.6 Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.</p>
DOK Levels:	1, 2
Achievement Level Descriptors:	
<p>RANGE Achievement Level Descriptor (Range ALD) Target J: Investigate patterns of association in bivariate data.</p>	<p>Level 1 Students should be able to investigate a scatter plot for clustering between two quantities and construct a scatter plot from given data. They should be able to construct a two-way frequency table of given categorical data.</p> <p>Level 2 Students should be able to investigate a scatter plot for positive, negative, and linear association and informally fit a line to data for a given a scatter plot that suggests a linear association. They should be able to calculate frequencies from categorical data in a two-way frequency table.</p> <p>Level 3 Students should be able to investigate a scatter plot for patterns such as outliers and nonlinear association. They should be able to write an equation for the trend line or line of best fit for a given scatter plot with a linear association. They should also be able to interpret and use relative frequencies from a two way table to describe possible association between two variables.</p> <p>Level 4 Students should be able to use scatter plots, trend lines, and associations between variables in two-way frequency tables to make predictions in real-world situations.</p>
Evidence Required:	<ol style="list-style-type: none"> 1. The student interprets patterns of association between two quantities in a scatter plot (clustering in reference to the line of best fit, positive or negative association, linear association, nonlinear association, and the effect of outliers) and interprets the slope and y-intercept in terms of the context. 2. The student identifies the slope (rate of change) and intercept (initial value) of a line suggested by examining bivariate measurement data in a scatter plot. 3. The student constructs and interprets a two-way table summarizing data on two categorical variables collected from the same subjects.

Grade 8 Mathematics Item Specification C1 TJ

	4. The student uses relative frequencies calculated for rows or columns to describe possible association between the two variables.
Allowable Response Types:	Matching Table; Multiple Choice
Allowable Stimulus Materials:	Scatter plot, two-way relative frequency table, raw data
Construct-Relevant Vocabulary:	cluster, data, frequency, initial value, line of best fit, trend line, linear extrapolation, linear association, negative association, outlier, positive association, rate of change, relative frequency, scale, scatter plot, slope, two-way relative frequency table, variable, x-axis, y-axis, x-intercept, y-intercept
Allowable Tools:	Calculator
Target-Specific Attributes	
Non-Targeted Constructs:	
Accessibility Concerns:	<p>Item writers should consider the following Language and Visual Element/Design guidelines¹ when developing items.</p> <p>Language Key Considerations:</p> <ul style="list-style-type: none"> • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references <p>Visual Elements/Design Key Considerations:</p> <ul style="list-style-type: none"> • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics <p>Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.²</p>
Development Notes:	Assessing the fit of a model (8.SP.A.2) will be assessed in Claim 4. Interpreting the slope and y-intercept in context (8.SP.A.3) will be assessed in Claims 2 and 4.

¹ For more information, refer to the General Accessibility Guidelines at:

<http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf>

² For more information about student accessibility resources and policies, refer to

http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1**Response Type:**
Matching Table**DOK Level 1****8.SP.A.1**

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities.

Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.A.2

Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

Evidence Required:

1. The student interprets patterns of association between two quantities in a scatter plot (clustering in reference to the line of best fit, positive or negative association, linear association,

Prompt Features: The student is prompted to determine whether statements about the data in a scatter plot are true.

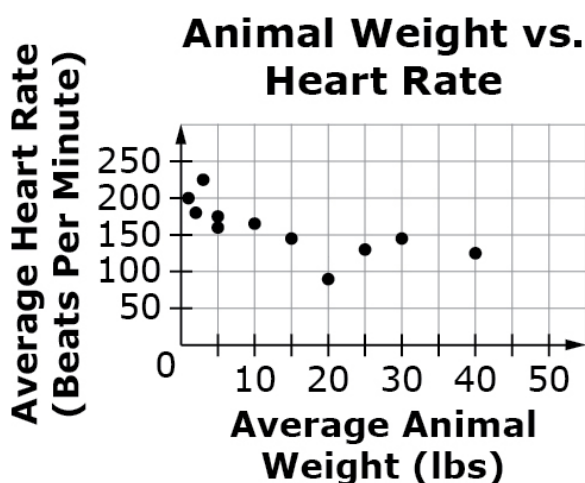
Stimulus Guidelines:

- Context should be familiar to students 13–15 years old.
- Scatter plot will have an informative title relevant to the situation.
- Axes will have informative titles relevant to the situation and appropriate interval scales.
- The data set may include clustering.
- Item difficulty can be adjusted via these example methods:
 - The association may be positive, negative, linear, or nonlinear.
 - There may be clustering, gaps, and outliers in the data.

TM1a

Stimulus: The student is presented with a situation that involves a relationship between two quantities and a scatter plot of measurements of those quantities with sufficient points to demonstrate a linear or nonlinear relationship.

Example Stem: This scatter plot shows the relationship between the average weight and average heart rate for 11 different animals.



Select True or False for each statement based on the scatter plot.

Statement	True	False
There is a positive association between average weight and average heart rate for animals.		
Animals with higher body weights tend to have lower heart rates than animals with lower body weights.		
For animals weighing 20 lbs or less, there is a linear association between average weight and average heart rate.		

Grade 8 Mathematics Item Specification C1 TJ

<p>nonlinear association, and the effect of outliers) and interprets the slope and y-intercept in terms of the context.</p> <p>Tools: Calculator</p>	<p>Rubric: (1 point) Student determines each statement as being either true or false (e.g., F, T, T) Each statement that interprets the scatter plot and may involve clustering in reference to the line of best fit, positive or negative associations, linear associations, nonlinear associations, or the effect of outliers.</p> <p>Response Type: Matching Table</p>
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Task Model 1

Response Type:
Matching Table

DOK Level 1

8.SP.A.1

Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.A.2

Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.A.3

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

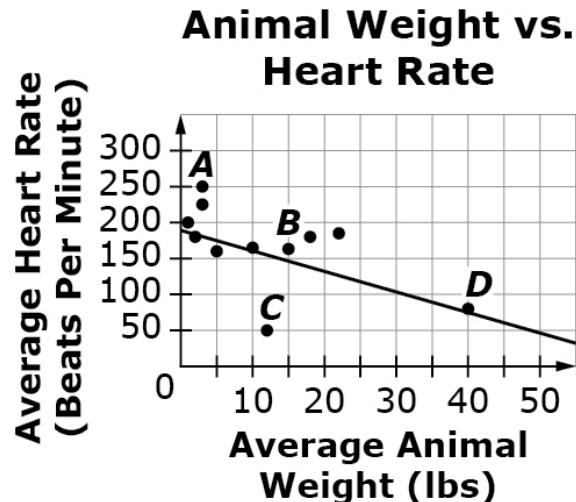
Evidence Required:

1. The student interprets patterns of

TM1b

Stimulus: The student is presented with a situation that involves a relationship between two quantities and a scatter plot of measurements of those quantities and a line purported to be a line of best fit.

Example Stem: This scatter plot shows the relationship between the average weight and average heart rate for 11 different animals. The line of best fit is shown on the graph.



Select True or False for each statement based on the graph.

Statement	True	False
The line of best fit provides a good estimate of an animal's average heart rate based on its weight for all animals.		
The y-intercept is at approximately (0, 185).		
Point D is one outlier because it is far away from the other data points.		

Rubric: (1 point) Student determines each statement as being either true or false (e.g., F, T, F) Each statement interprets the scatter plot and may involve clustering in reference to the line of best fit, positive or negative associations, linear associations, nonlinear associations, the effect of outliers, the identification or the interpretation of the slope or y-intercept in terms of the context.

Response Type: Matching Table

<p>association between two quantities in a scatter plot (clustering in reference to the line of best fit, positive or negative association, linear association, nonlinear association, and the effect of outliers) and interprets the slope and y-intercept in terms of the context.</p> <p>Tools: Calculator</p>	
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Task Model 2

Response Type:
Multiple Choice,
single correct
response

DOK Level 2

8.SP.3

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

Evidence Required:

2. The student identifies the slope (rate of change) and intercept (initial value) of a line suggested by examining bivariate measurement data in a scatter plot.

Tools: Calculator

Version 3 Update:

Revised TM2 and changed from equation/numeric response type to multiple choice. Added a second example stem.

Prompt Features: The student is prompted to interpret the slope and y-intercept of the line of best fit on a scatter plot.

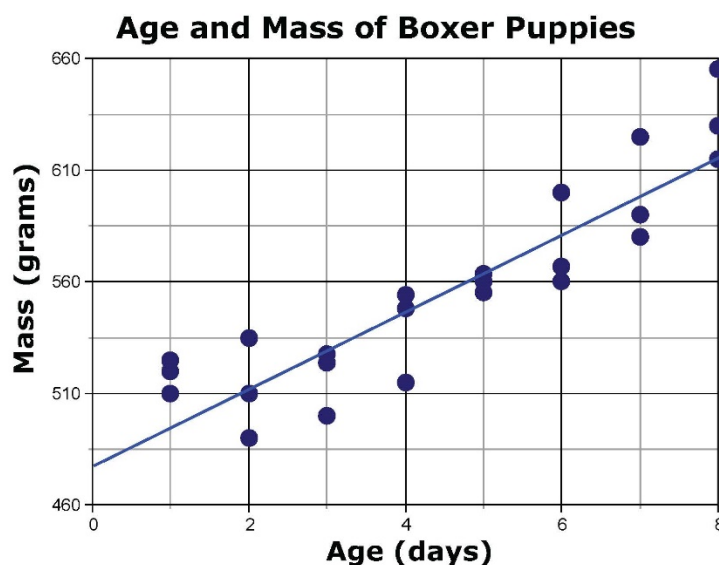
Stimulus Guidelines:

- Context should be familiar to students 13–15 years old.
- Scatter plot will have an informative title relevant to the situation.
- Axes will have informative titles relevant to the situation and appropriate interval scales.
- The data set may include clustering.
- Item difficulty can be adjusted via these example methods:
 - The association may be positive, negative, linear, or nonlinear.
 - The data set may reflect an explicit or implicit linear relationship or explicit or implicit nonlinear relationship.
 - There may be clustering, gaps, and outliers in the data.

TM2a

Stimulus: The student is presented with a situation that involves a relationship between two quantities and a scatter plot measurements of those two quantities with sufficient points to demonstrate a linear relationship. The graph provides the line of best fit.

Example Stem 1: Every boxer puppy in a litter is weighed each day. The scatter plot shows the age and mass recorded at each weighing.



The line of best fit has equation $y = a + bx$, where a and b are constants. What does the y-intercept tell you about the puppies in the litter?

- A. The predicted change in mass of a puppy each day.
- B. The predicted mass of a puppy at birth.
- C. The number of puppies born on day 0.
- D. The mass of the entire litter of puppies.

Task Model 2

Response Type:
Multiple Choice,
single correct
response

DOK Level 2

8.SP.3

Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

Evidence Required:

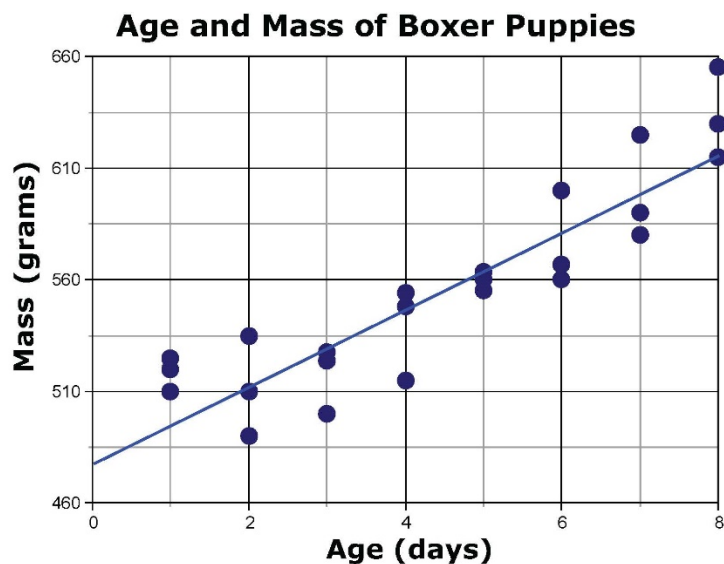
2. The student identifies the slope (rate of change) and intercept (initial value) of a line suggested by examining bivariate measurement data in a scatter plot.

Tools: Calculator

Version 3 Update:

Revised TM2 and changed from equation/numeric response type to multiple choice. Added a second example stem.

Example Stem 2: Each puppy in a litter is weighed each day. The scatter plot shows the age and mass recorded at each weighing.



The line of best fit is shown on the scatter plot. What does the slope of the line tell you about the puppies in the litter?

- A. The predicted change in mass of a puppy each day.
- B. The predicted mass of a puppy at birth.
- C. The number of puppies born on day 0.
- D. The mass of the entire litter of puppies.

Rubric: (1 point) Student selects the correct answer (e.g., B; A).

Response Type: Multiple Choice, single correct response

<p>Task Model 3</p> <p>Response Type: Multiple Choice, single correct response</p> <p>DOK Level 2</p> <p>8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.</p> <p>Evidence Required: 3. The student constructs and interprets a two-way table summarizing data on two categorical variables collected from the same subjects.</p> <p>Tools: Calculator</p> <p>Version 3 Update: Revised TM3 and changed from fill-in table response type to multiple choice.</p>	<p>Prompt Features: The student is given a two-way table for two categorical variables for the same population and asked to interpret it.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none">Context should be familiar to students 13–15 years old. <p>TM3 Stimulus: The student is presented with a situation that involves two categorical variables and is asked to interpret the information provided in the table.</p> <p>Example Stem 1: All 8th-grade students at a school answered Yes or No to the two survey questions shown.</p> <ul style="list-style-type: none">Do you have a cell phone? Yes NoDo you have an MP3 player? Yes No <p>The results of the survey are shown in the table.</p> <table><tr><th></th><th>MP3 Player</th><th>No MP3 Player</th><th>Total</th></tr><tr><td>Cell Phone</td><td>58</td><td>122</td><td>180</td></tr><tr><td>No Cell Phone</td><td>30</td><td>65</td><td>95</td></tr><tr><td>Total</td><td>88</td><td>187</td><td>275</td></tr></table> <p>What percentage of the students have both a cell phone and an MP3 player?</p> <ul style="list-style-type: none">A. 21%B. 32%C. 66%D. 68% <p>Example Stem 2: A company surveyed both adults and children about whether or not they liked a particular game. The survey results are shown in the table.</p> <table><tr><th></th><th>Liked the game</th><th>Did not like the game</th><th>Total</th></tr><tr><td>Adults</td><td>28</td><td>20</td><td>48</td></tr><tr><td>Children</td><td>54</td><td>98</td><td>152</td></tr><tr><td>Total</td><td>82</td><td>118</td><td>200</td></tr></table> <p>Which of the following correctly compares the proportion of adults who liked the game with the proportion of children who liked the game?</p> <ul style="list-style-type: none">A. They are approximately the same.B. The proportion of adults who liked the game is <u>greater than</u> the proportion of children who liked the game.C. The proportion of adults who liked the game is <u>less than</u> the proportion of children who liked the game.D. It is not possible to compare these proportions with the information given. <p>Rubric: (1 point) Student selects the correct answer (e.g., A, B).</p> <p>Response Type: Multiple Choice, single correct response</p>		MP3 Player	No MP3 Player	Total	Cell Phone	58	122	180	No Cell Phone	30	65	95	Total	88	187	275		Liked the game	Did not like the game	Total	Adults	28	20	48	Children	54	98	152	Total	82	118	200
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<p>Task Model 4</p> <p>Response Type: Multiple Choice, single correct</p> <p>DOK Level 2</p> <p>8.SP.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.</p> <p>Evidence Required: 4. The student uses relative frequencies calculated for rows or columns to describe possible association between the two variables.</p> <p>Tools: Calculator</p> <p>Version 3 Update: Added new evidence required statement and TM4.</p>	<p>Prompt Features: The student is presented with a situation that involves two categorical variables and asked if there is evidence of an association between them or to identify evidence of an association that exists.</p> <p>Stimulus Guidelines:</p> <ul style="list-style-type: none">Context should be familiar to students 13–15 years old. <p>TM4a Stimulus: The student is presented with a situation that involves two categorical variables.</p> <p>Example Stem: All 8th-grade students at a school answered Yes or No to the two survey questions shown.</p> <ul style="list-style-type: none">Do you have a cell phone? Yes NoDo you have an MP3 player? Yes No <p>The results of the survey are shown in the table.</p> <table><tr><th></th><th>MP3 Player</th><th>No MP3 Player</th><th>Total</th></tr><tr><td>Cell Phone</td><td>58</td><td>122</td><td>180</td></tr><tr><td>No Cell Phone</td><td>30</td><td>65</td><td>95</td></tr><tr><td>Total</td><td>88</td><td>187</td><td>275</td></tr></table> <p>Is there an association between owning a cell phone and owning an MP3 Player for the students at this school?</p> <ul style="list-style-type: none">A. Yes, because more than half of the students own a cell phone but less than half own an MP3 player.B. Yes, because the proportion of students who own an MP3 player is almost the same for students who own a cell phone and for students who do not.C. No, because more than half of the students own a cell phone but less than half own an MP3 player.D. No, because the proportion of students who own an MP3 player is almost the same for students who own a cell phone and for students who do not. <p>Rubric: (1 point) Student selects the correct percent (e.g., D).</p> <p>Response Type: Multiple Choice, single correct response</p>		MP3 Player	No MP3 Player	Total	Cell Phone	58	122	180	No Cell Phone	30	65	95	Total	88	187	275
	MP3 Player	No MP3 Player	Total														
Cell Phone	58	122	180														
No Cell Phone	30	65	95														
Total	88	187	275														

Task Model 4

Response Type:
Multiple Choice,
single correct

DOK Level 2

8.SP.4

Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.

Evidence Required:

4. The student uses relative frequencies calculated for rows or columns to describe possible association between the two variables.

Tools: Calculator

Version 3 Update:
Added new evidence required statement and TM4.

TM4b

Stimulus: The student is presented with a situation that involves two categorical variables and asked if there is evidence of an association between them or to identify evidence of an association that exists.

Example Stem: A coach of a cross country team asked all 200 of the runners who ran at a meet two questions:

Did you get less than 8 hours of sleep last night?	Yes	No
Did you achieve a personal record in this meet?	Yes	No

A summary of the data is shown in the table.

	8 or more hours of sleep	Less than 8 hours of sleep	Total
Personal Record	28	20	48
No Personal Record	54	98	152
Total	82	118	200

The coach saw an association between the amount of sleep and achieving a personal record for the runners. Which statement provides evidence for this association?

- A. About 34% of the runners who got more than 8 hours of sleep achieved a personal record, but only 17% of those who got less than 8 hours of sleep did.
- B. More than 50% of the runners who achieved a personal record got 8 or more hours of sleep.
- C. Only 25% of the runners achieved a personal record at the meet.
- D. There is no evidence for an association.

Rubric: (1 point) Student selects the correct percent (e.g., A).

Response Type: Multiple Choice, single correct response