Claim 1: Concepts and Procedures
Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.

Content Domain: **Numbers and Operations—Fractions**

Target F [m]: Extend understanding of fraction equivalence and ordering. (DOK 1, 2)

Tasks for this target will ask students to recognize and generate equivalent fractions or compare fractions with different numerators and different denominators, sometimes using <, =, and >. These may include the use of visual fraction models or number lines to tap student understanding of equivalence and relative size with respect to benchmarks, such as 1/2.

Standards: 4.NF.A, 4.NF.A.1, 4.NF.A.2	**4.NF.A Extend understanding of fraction equivalence and ordering.** **4.NF.A.1** Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **4.NF.A.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling: 3.NF.A, 3.NF.A.3, 3.NF.A.3a, 3.NF.A.3b, 3.NF.A.3d 5.NF.A, 5.NF.A.1, 5.NF.A.2	**Related Grade 3 Standards** **3.NF.A Develop understanding of fractions as numbers.** **3.NF.A.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. **a**. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. **b**. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model. **d.** Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

	Related Grade 5 Standards **5.NF.A Use equivalent fractions as a strategy to add and subtract fractions.** **5.NF.A.1** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)* **5.NF.A.2** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.*
DOK Levels:	1, 2
Achievement Level Descriptors:	
RANGE Achievement Level Descriptor (Range ALD) Target F: Extend understanding of fraction equivalence and ordering.	**Level 1** Students should be able to recognize that fraction comparisons are valid only when the two fractions are referring to the same whole.
	Level 2 Students should be able to compare two fractions with different numerators and different denominators using <, >, and = by comparing to a benchmark fraction such as 1/2 and recognize equivalent fractions using visual models.
	Level 3 Students should be able to extend understanding to compare two fractions with different numerators and different denominators using <, >, and = by creating common denominators or numerators and recognize and generate equivalent fractions using visual models.
	Level 4 Students should be able to extend understanding to compare two fractions with different numerators and different denominators using <, >, and = and justify the conclusions using a visual fraction model.
Evidence Required:	1. The student recognizes when two or more fractions are equivalent. 2. The student generates equivalent fractions given an initial fraction or fraction model. 3. The student uses the symbols <, >, and = to compare fractions with different numerators and different denominators.
Allowable Response Types:	Matching Tables; Equation/Numeric; Multiple choice, multiple correct responses; Hot Spot
Allowable Stimulus Materials:	<, >, and = symbols, number lines, parts of whole visual models, parts of set visual models, tables
Construct-Relevant Vocabulary:	fraction, equivalent, divide, equal to, greater than, less than, digits, numerator, denominator
Allowable Tools:	None

Target-Specific Attributes:	The majority of items in this target should follow the CCSS limitations on denominators allowed at Grade 4 (2, 3, 4, 5, 6, 8, 10, 12, and 100). For the purposes of adaptive testing, however, some items may use denominators appropriate to 5th grade (multiples of 2, 3, 5, and/or 7 that are less than or equal to 100).
Non-Targeted Constructs:	None
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines[1] when developing items. Language Key Considerations: • Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the understanding of the context • Avoid sentences with multiple clauses • Use vocabulary that is at or below grade level • Avoid ambiguous or obscure words, idioms, jargon, unusual names and references Visual Elements/Design Key Considerations: • Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context • Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary • Avoid crowding of details and graphics Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models. As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.[2]
Development Notes:	Explaining why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ (CCSS 4.NF.A.1), will be assessed in Claim 3. Recognizing that comparisons are valid only when the two fractions refer to the same whole (CCSS 4.NF.A.2) will be assessed in Claim 3. Justifying the comparison of fractions (CCSS 4.NF.A.2) will be assessed in Claim 3.

[1] For more information, refer to the General Accessibility Guidelines at: http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf

[2] For more information about student accessibility resources and policies, refer to http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf

Task Model 1a

Response Type: Matching Tables

DOK Level 1

4.NF.A.1
Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Evidence Required:
1. The student recognizes when two or more fractions are equivalent.

Tools: None

Prompt Features: The student is prompted to identify equivalent fractions.

Stimulus Guidelines:

- The fractions in the table have different denominators than the given fraction.
- Item difficulty can be adjusted via these example methods:
  - Presenting fractions that are less than or greater than 1
  - Using denominators that are multiples of 2, 3, 4, 5, 6, 8, 10, 12, or 100, but not actually those numbers (e.g., 9, 15, or 18)

TM1a
Stimulus: The student is presented with a visual fraction model in the form $\frac{a}{b}$.

Example Stem: Figure A has $\frac{2}{3}$ of its whole shaded gray.

Figure A

Decide whether each fraction is equal to $\frac{2}{3}$. Select Yes or No for each fraction.

	Yes	No
$\frac{4}{6}$		
$\frac{1}{2}$		
$\frac{8}{12}$		

Rubric: (1 point) The student correctly identifies all of the fractions as equivalent or not equivalent (e.g., Y, N, Y).

Response Type: Matching Tables

Task Model 1b

Response Type: Matching Tables

DOK Level 1

4.NF.A.1
Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Evidence Required:
1. The student recognizes when two or more fractions are equivalent.

Tools: None

Prompt Feature: The student is prompted to identify equivalent fractions.

Stimulus Guidelines:

- Equations show pairs of fractions with different numerators and denominators.
- At least one fraction of each pair should have a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100.
- Item difficulty can be adjusted via these example methods:
  - Location of the fraction with a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100 (left or right side of equation)
  - Using denominators that are multiples of 2, 3, 4, 5, 6, 8, 10, 12, or 100, but not actually those numbers (e.g., 9, 15, or 18)
  - Presenting fractions that are less than or greater than 1

TM1b
Stimulus: The student is presented with pairs of fractions in numeric form in the answer choices.

Example Stem: Select True if the equation is true. Select False if the equation is **not** true.

	True	False
$\frac{4}{6} = \frac{8}{12}$		
$\frac{50}{100} = \frac{3}{4}$		
$\frac{6}{8} = \frac{75}{100}$		

Rubric: (1 point) The student correctly identifies all fraction equivalencies as True or False (e.g., T, F, T).

Response Type: Matching Tables

Task Model 1c

Response Type: Matching Tables

DOK Level 1

4.NF.A.1
Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Evidence Required:
1. The student recognizes when two or more fractions are equivalent.

Tools: None

Prompt Feature: The student is prompted to identify equivalent fractions.

Stimulus Guidelines:

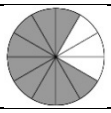
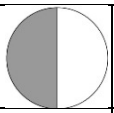
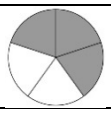
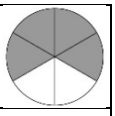
- All fractions used should have at least one equivalent fraction (e.g., there should be no fractions that do not have any matches).
- At least one fraction of each match should have a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100.
- Item difficulty can be adjusted via these example methods:
  - Location of the fraction with a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100 (along left side or top)
  - Using denominators that are multiples of 2, 3, 4, 5, 6, 8, 10, 12, or 100, but not actually those numbers (e.g., 9, 15, or 18)
  - Having more than one match per fraction
  - Presenting fractions that are less than or greater than 1

TM1c

Stimulus: The student is presented with four visual fraction models and four fractions in numeric form.

Example Stem: A fraction of the whole is shaded in each model.

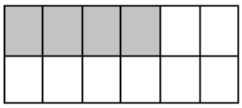
Click in the chart to match each fraction to the shaded part of the model that shows an equivalent fraction.

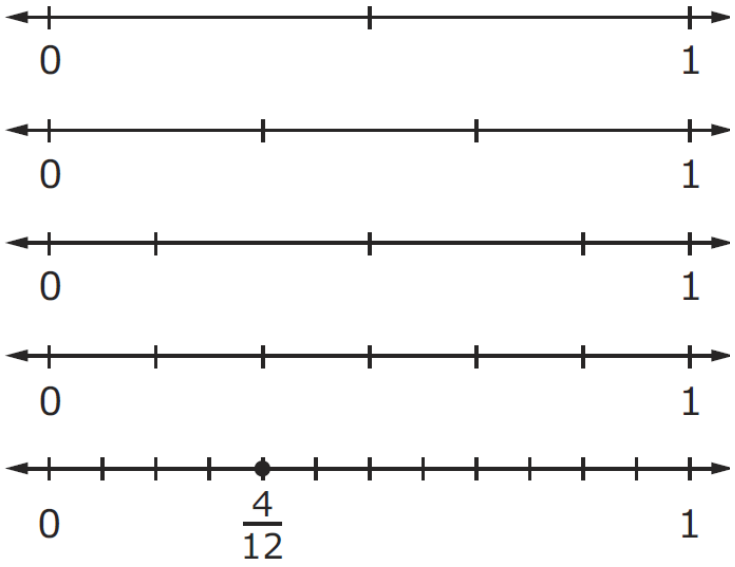
				
$\frac{2}{3}$				
$\frac{3}{4}$				
$\frac{4}{8}$				
$\frac{6}{10}$				

Rubric: (1 point) The student correctly matches all fractions to its model (e.g., $\frac{3}{4} \rightarrow$  , $\frac{4}{8} \rightarrow$  , $\frac{6}{10} \rightarrow$  , $\frac{2}{3} \rightarrow$ )

Response Type: Matching Tables

Task Model 1d **Response Type: Multiple Choice, multiple correct responses** **DOK Level 1** **4.NF.A.1** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **Evidence Required:** 1. The student recognizes when two or more fractions are equivalent. **Tools:** None **Version 3 Update:** Added new TM1d.	**TM1d** **Stimulus:** The student is presented with six fractions in numeric form. **Example Stem:** Select **all** fractions that are equal to $\frac{3}{4}$. A. $\frac{1}{2}$ B. $\frac{3}{5}$ C. $\frac{4}{6}$ D. $\frac{6}{8}$ E. $\frac{6}{10}$ F. $\frac{9}{12}$ **Rubric:** (1 point) The student selects all of the equivalent fractions (e.g., D, F). **Response Type:** Multiple choice, multiple correct responses

Task Model 2a **Response Type: Equation / Numeric** **DOK Level 1** **4.NF.A.1** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **Evidence Required:** 2. The student generates equivalent fractions given an initial fraction or fraction model. **Tools:** None	**Prompt Features:** The student is prompted to enter an equivalent fraction. **Stimulus Guidelines:** • The given fraction should have a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100. • Fraction model must represent the given fraction (total shaded sections = numerator, total sections = denominator). • Item difficulty can be adjusted via these example methods: o Location of the shaded sections (e.g., all connected or scattered apart from each other) o Student familiarity with the denominator used o Presenting fractions that are less than or greater than 1 o Presenting fractions greater than 1 as improper fractions or mixed numbers **TM2a** **Stimulus:** The student is presented with a visual fraction model. **Example Stem:** Figure A has $\frac{4}{12}$ of its whole shaded.  **Figure A** Enter **another** fraction that is equal to $\frac{4}{12}$. **Rubric:** (1 point) The student enters a fraction equivalent to the given fraction (e.g., $\frac{1}{3}$; $\frac{8}{24}$, etc). **Scoring Note:** The fraction given in the stem (e.g., $\frac{4}{12}$) will not be accepted as a correct answer. **Response Type:** Equation/Numeric

Task Model 2b **Response Type: Equation / Numeric** **DOK Level 1** **4.NF.A.1** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **Evidence Required:** 2. The student generates equivalent fractions given an initial fraction or fraction model. **Tools:** None **Version 3 Update:** Added a new example stem for TM2b.	**TM2b** **Stimulus:** The student is presented with a fraction in numeric form, with or without a series of number lines with one fraction labeled. **Example Stem 1:** Enter **another** fraction that is equivalent to $\frac{4}{12}$. **Example Stem 2:** Figure B shows several number lines that divide 1 into equal parts. **Figure B**  Enter **another** fraction that is equal to $\frac{4}{12}$. **Rubric:** (1 point) The student enters a fraction equivalent to the given fraction (e.g., $\frac{1}{3}$ or $\frac{2}{6}$ or other equivalent fraction). **Scoring Note:** The fraction given in the stem (e.g., $\frac{4}{12}$) will not be accepted as a correct answer. **Response Type:** Equation/Numeric

Task Model 2c **Response Type:** **Hot Spot** **DOK Level 2** **4.NF.A.1** Explain why a fraction *a*/*b* is equivalent to a fraction (*n* × *a*)/(*n* × *b*) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. **Evidence Required:** 2. The student generates equivalent fractions given an initial fraction or fraction model. **Tools:** None **Accessibility Note:** Hot Spot items are not currently able to be Brailled. Minimize the number of items developed to this TM using Hot Spot.	**Prompt Features:** The student is prompted to generate a fraction model that is equivalent to a given fraction or fraction model. **Stimulus Guidelines:** • The given fraction should have a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100. • The number of sections of the fraction model should be a multiple or factor of the denominator of the given fraction (e.g., if given fraction is $\frac{4}{6}$, fraction model could be in thirds or twelfths). • Item difficulty can be adjusted via these example methods: o Using a fraction model with the number of sections other than 2, 3, 4, 5, 6, 8, 10, 12, or 100 o Student familiarity with the denominator used o Presenting fractions that are less than or greater than 1 o Presenting fractions greater than 1 as improper fractions or mixed numbers **TM2c** **Stimulus:** The student is presented with a fraction in numeric form. **Example Stem:** Click the spaces of the model to shade $\frac{3}{6}$ of Figure A.  **Figure A** **Rubric:** (1 point) The student builds a model of an equivalent fraction (e.g., $\frac{2}{4}$). **Response Type:** Hot Spot

Task Model 3a

Response Type: Matching Tables

DOK Level 2

4.NF.A.2
Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Evidence Required:
3. The student uses the symbols <, >, and = to compare fractions with different numerators and different denominators.

Tools: None

Version 3 Update:
Added more example methods for varying the item difficulty to the stimulus guidelines.

Prompt Feature: The student is prompted to compare two fractions.

Stimulus Guidelines:

- At least one of each pair of the given fractions should have a denominator of 2, 3, 4, 5, 6, 8, 10, 12, or 100.
- Item difficulty can be adjusted via these example methods:
  - Selecting fractions that are equivalent; have same denominator or same numerator; fractions that are not related such as 4/5 and 2/3
  - Selecting fractions that are close to benchmarks of 0, 1/2, or 1
  - Selecting fractions that are not near an easily recognized benchmark or are closer in value
  - Student familiarity with the denominator used
  - Presenting fractions that are less than or greater than 1
  - Presenting fractions greater than 1 as improper fractions or mixed numbers

TM3a
Stimulus: The student is presented with three fraction inequalities that compare two fractions each.

Example Stem: Select True if the comparison is true. Select False if the comparison is **not** true.

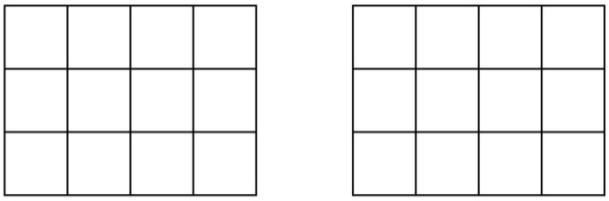
	True	False
$\frac{1}{4} < \frac{2}{12}$		
$\frac{2}{10} > \frac{3}{5}$		
$\frac{4}{6} > \frac{5}{12}$		

Rubric: (1 point) The student correctly identifies three fraction comparisons as either true or false (e.g., FFT).

Response Type: Matching Tables

Task Model 3b **Response Type: Matching Table** **DOK Level 2** **4.NF.A.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **Evidence Required:** 3. The student uses the symbols <, >, and = to compare fractions with different numerators and different denominators. **Tools:** None **Version 3 Update:** Changed TM3b from an equation/numeric response type to a matching table response type. Updated the stimulus and stem to match the new format.	**Prompt Feature:** The student is prompted to compare two fractions. **Stimulus Guidelines:** Same as for TM3a. **TM3b** **Stimulus:** The student is presented with two pairs of fractions and directed to compare them using (<, >, or =). **Example Stem:** Select the symbol (<, >, or =) that correctly compares each pair of numbers. **Rubric:** (1 point) The student selects the correct symbols (e.g., =, <). **Response Type:** Matching Table

	<	>	=
$\frac{2}{8}$ ☐ $\frac{1}{4}$			
$\frac{3}{5}$ ☐ $\frac{7}{8}$			

Task Model 3c **Response Type:** **Hot Spot** **DOK Level 2** **4.NF.A.2** Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model. **Evidence Required:** 3. The student uses the symbols <, >, and = to compare fractions with different numerators and different denominators. **Tools:** None **Accessibility Note:** Hot Spot items are not currently able to be Brailled. Minimize the number of items developed to this TM.	**Prompt Features:** The student is prompted to compare fractions and justify the comparison with visual models. **Stimulus Guidelines:** • Fraction models should reflect a common multiple of the denominators of the fractions (not necessarily the least common denominator). **TM3c** **Stimulus:** The student is presented with two fractions and two blank fraction models. **Example Stem:** • Click on the squares in the rectangles that are needed to represent $\frac{4}{6}$ and $\frac{2}{4}$, as labeled below each large rectangle. • Choose the correct symbol to compare the fractions. Each large rectangle represents one whole.  $\frac{4}{6}$ < > = $\frac{2}{4}$ **Rubric:** (2 points) The student correctly shades the fraction models and chooses the correct comparison symbol (e.g., the left model shows $\frac{8}{12}$, the right model shows $\frac{6}{12}$, and the symbol selected is >). (1 point) Partial credit is possible for either shading the fraction models correctly, or for choosing the correct comparison symbol. **Response Type:** Hot Spot