

<b>Claim 1: Concepts and Procedures</b> Students can explain and apply mathematical concepts and carry out mathematical procedures with precision and fluency.	
Content Domain: <b>Measurement and Data</b>	
<b>Target K [a]:</b> Geometric measurement: understand concepts of angle and measure angles. (DOK 1, 2)  Tasks for this target will ask students to construct and measure angles using a protractor; to provide multiple ways to decompose a larger angle into two or more smaller angles that have the same sum as the original angle; and to determine an unknown angle measure in a diagram. Some tasks will connect the angle measure back to the number of adjacent one-degree angles that comprise the whole.	
Standards: 4.MD.C, 4.MD.C.5, 4.MD.C.5a, 4.MD.C.5b, 4.MD.C.6, 4.MD.C.7	<b>4.MD.C Geometric measurement: understand concepts of angle and measure angles.</b>  <b>4.MD.C.5</b> Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: <ol style="list-style-type: none"> <li>An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through <math>\frac{1}{360}</math> of a circle is called a "one-degree angle," and can be used to measure angles.</li> <li>An angle that turns through <math>n</math> one-degree angles is said to have an angle measure of <math>n</math> degrees.</li> </ol> <b>4.MD.C.6</b> Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.  <b>4.MD.C.7</b> Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.
Related Below-Grade and Above-Grade Standards for Purposes of Planning for Vertical Scaling:  None	<b>Related Grade 3 Standards</b>  None  <b>Related Grade 5 Standards</b>  None
DOK Levels:	1, 2

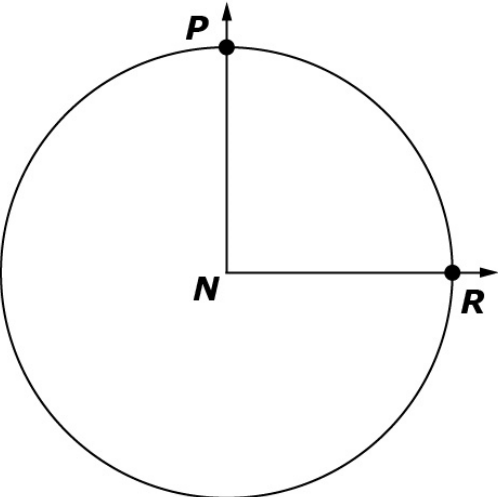
<b>Achievement Level Descriptors:</b>	
<b>RANGE Achievement Level Descriptor (Range ALD)</b> Target K: Geometric measurement: understand concepts of angles and measure angles.	<b>Level 1</b> No Descriptor
	<b>Level 2</b> Students should be able to recognize whole-number degrees on a protractor and measure angles in whole-number degrees using a protractor.
	<b>Level 3</b> Students should be able to construct angles in whole-number degrees using a protractor, use understanding of angle concepts to decompose a larger angle with two or more smaller angles that have the same sum as the original, and determine an unknown angle measure in a diagram.
	<b>Level 4</b> Students should be able to solve addition and subtraction problems to find unknown angles on a diagram in problems by using an equation with a symbol for the unknown angle measure.
<b>Evidence Required:</b>	1. The student relates the concept of an angle to the fraction of a circular arc between two points on a circle. 2. The student uses a protractor to measure angles (composed of one-degree angles) and construct angles to whole-number degrees. 3. The student decomposes an angle into smaller non-overlapping parts and adds the measures of these smaller parts to find the measure of the whole angle. 4. The student solves addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.
Allowable Response Types:	Equation/Numeric; Graphing; Drag and Drop
Allowable Stimulus Materials:	graphics of angles, turns, and rotations; protractors
Construct-Relevant Vocabulary:	protractor, angle, ray, intersect, one-degree angle, vertex, ray
Allowable Tools:	protractor
Target-Specific Attributes:	Benchmark angles are 30°, 45°, 60°, 90°, 180°
Non-Targeted Constructs:	None
Accessibility Guidance:	Item writers should consider the following Language and Visual Element/Design guidelines <sup>1</sup> when developing items.  Language Key Considerations: <ul style="list-style-type: none"> <li>Use simple, clear, and easy-to-understand language needed to assess the construct or aid in the</li> </ul>

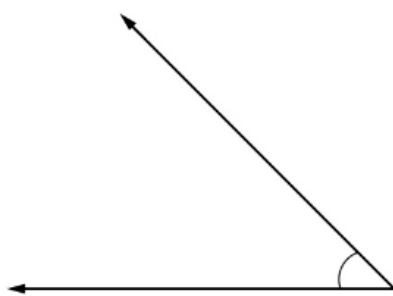
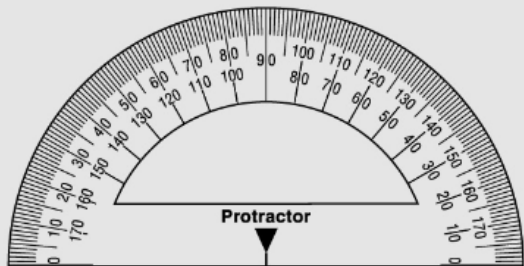
<sup>1</sup> For more information, refer to the General Accessibility Guidelines at:

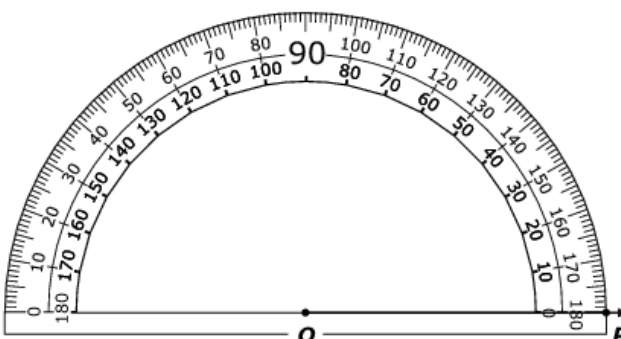
<http://www.smarterbalanced.org/wordpress/wp-content/uploads/2012/05/TaskItemSpecifications/Guidelines/AccessibilityandAccommodations/GeneralAccessibilityGuidelines.pdf>

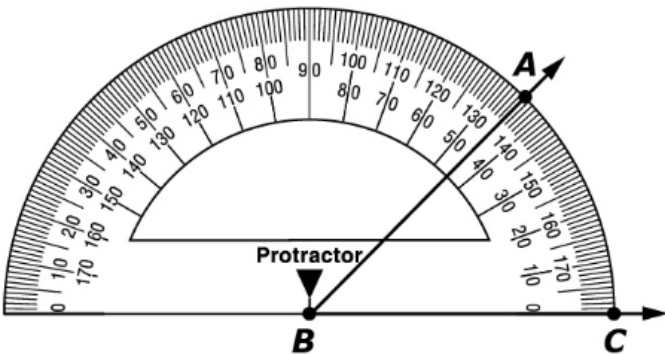
	<p>understanding of the context</p> <ul style="list-style-type: none"> <li>• Avoid sentences with multiple clauses</li> <li>• Use vocabulary that is at or below grade level</li> <li>• Avoid ambiguous or obscure words, idioms, jargon, unusual names and references</li> </ul> <p>Visual Elements/Design Key Considerations:</p> <ul style="list-style-type: none"> <li>• Include visual elements only if the graphic is needed to assess the construct or it aids in the understanding of the context</li> <li>• Use the simplest graphic possible with the greatest degree of contrast, and include clear, concise labels where necessary</li> <li>• Avoid crowding of details and graphics</li> </ul> <p>Items are selected for a student's test according to the blueprint, which selects items based on Claims and targets, not task models.</p> <p>As such, careful consideration is given to making sure fully accessible items are available to cover the content of every Claim and target, even if some item formats are not fully accessible using current technology.<sup>2</sup></p>
Development Notes:	<p>Representing the addition or subtraction of angle measures with an equation will be assessed in Claim 4.</p> <p>Identifying angles of specified measures will be assessed at Claim 2.</p>

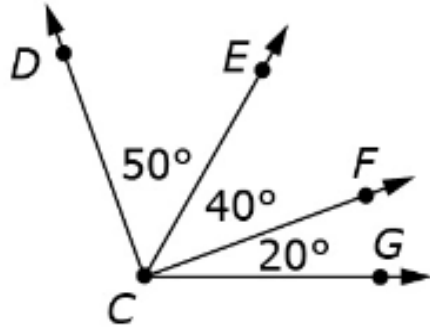
<sup>2</sup> For more information about student accessibility resources and policies, refer to [http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced\\_Guidelines.pdf](http://www.smarterbalanced.org/wordpress/wp-content/uploads/2014/08/SmarterBalanced_Guidelines.pdf)

<p><b>Task Model 1</b></p> <p><b>Response Type:</b> <b>Equation/Numeric</b></p> <p><b>DOK Level 2</b></p> <p><b>4.MD.C.5</b> Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:  <b>a.</b> An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through <math>\frac{1}{360}</math> of a circle is called a "one-degree angle," and can be used to measure angles.  <b>b.</b> An angle that turns through <math>n</math> one-degree angles is said to have an angle measure of <math>n</math> degrees.</p> <p><b>Evidence Required:</b> 1. The student relates the concept of an angle to the fraction of a circular arc between two points on a circle.</p> <p><b>Tools:</b> None</p>	<p><b>Prompt Features:</b> The student is prompted to write the measure of an angle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where two rays intersect the circle.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> <li>Whether the angle presented represents a benchmark arc size (e.g., <math>\frac{1}{4}</math> of a circle is easier than <math>\frac{1}{5}</math> of a circle)</li> <li>Whether the item presents a real-world context or mathematical context</li> </ul> </li> </ul> <p><b>TM1</b>  <b>Stimulus:</b> The student is presented with an angle superimposed on a circle with its vertex at the center of the circle and the fraction of a circular arc that it represents.</p> <p><b>Example Stem:</b> The vertex of <math>\angle PNR</math> is at the center of the circle. The circular arc between Point <math>P</math> and Point <math>R</math> is <math>\frac{1}{4}</math> of the circle.</p>  <p>Enter the measure, in degrees, of <math>\angle PNR</math>.</p> <p><b>Rubric:</b> The student enters the correct number of degrees (e.g., 90).</p> <p><b>Response Type:</b> Equation/Numeric</p>
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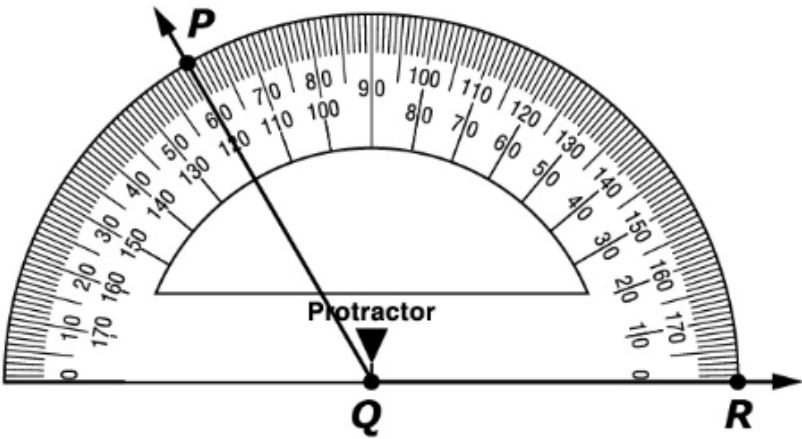
<p><b>Task Model 2a</b></p> <p><b>Response Type:</b> Drag and Drop</p> <p><b>DOK Level 1</b></p> <p><b>4.MD.C.6</b> Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</p> <p><b>Evidence Required:</b> 2. The student uses a protractor to measure angles (composed of one-degree angles) and construct angles to whole-number degrees.</p> <p><b>Tools:</b> Protractor</p> <p><b>Accessibility Note:</b> Drag and Drop items are not currently able to be Brailled. Minimize the number of items developed to this TM.</p>	<p><b>Prompt Features:</b> The student is prompted to measure an angle using a protractor.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>A protractor must be present in the palette.</li> <li>Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> <li>The orientation of the angle presented (e.g., with a horizontal base and opening up to the left)</li> <li>Whether the angle presented is a benchmark (i.e., <math>30^\circ</math>, <math>45^\circ</math>, <math>90^\circ</math>, or <math>180^\circ</math>) or non-benchmark angle</li> </ul> </li> </ul> <p><b>TM2a</b></p> <p><b>Stimulus:</b> The student is presented with an image of an angle.</p> <p><b>Example Stem:</b></p> <ul style="list-style-type: none"> <li>Use the protractor to measure the angle.</li> <li>Then drag the numbers into the box to enter the measure of the angle, in degrees.</li> </ul> <div data-bbox="568 882 1339 1176">  <div data-bbox="1153 1060 1339 1165" style="border: 1px solid black; width: 115px; height: 50px; display: flex; align-items: center; justify-content: center;"> <span style="font-size: 20px;">°</span> </div> </div> <div data-bbox="479 1218 1412 1606" style="background-color: #f0f0f0; padding: 10px; margin-top: 20px;"> <div style="display: flex; align-items: center;">  <table data-bbox="1169 1312 1315 1575" style="margin-left: 20px;"> <tbody> <tr><td>0</td><td>5</td></tr> <tr><td>1</td><td>6</td></tr> <tr><td>2</td><td>7</td></tr> <tr><td>3</td><td>8</td></tr> <tr><td>4</td><td>9</td></tr> </tbody> </table> </div> </div> <p><b>Rubric:</b> (1 point) The student enters the correct number of degrees in the angle (e.g., 45).</p> <p><b>Response Type:</b> Drag and Drop</p>	0	5	1	6	2	7	3	8	4	9
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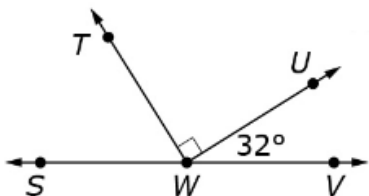
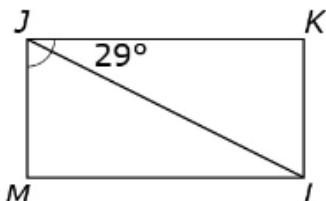
<p><b>Task Model 2b</b></p> <p><b>Response Type:</b> <b>Graphing</b></p> <p><b>DOK Level 2</b></p> <p><b>4.MD.C.6</b> Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</p> <p><b>Evidence Required:</b> 2. The student uses a protractor to measure angles (composed of one-degree angles) and construct angles to whole-number degrees.</p> <p><b>Tools:</b> Protractor</p> <p><b>Accessibility Note:</b> Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.</p>	<p><b>Prompt Features:</b> The student uses a protractor to construct an angle of a given measure.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> <li>The orientation of the ray that is presented (e.g., horizontal and pointing to the left)</li> <li>Whether the student is prompted to draw a benchmark (i.e., <math>30^\circ</math>, <math>45^\circ</math>, <math>90^\circ</math>, or <math>180^\circ</math>) or non-benchmark angle</li> </ul> </li> </ul> <p><b>TM2b</b></p> <p><b>Stimulus:</b> The student is presented with an angle measure and instructions to generate the angle.</p> <p><b>Example Stem:</b> Use the Add Arrow tool to draw a <math>45^\circ</math> angle that has ray <math>QP</math> as one of its sides.</p>  <p><b>Rubric:</b> (1 point) The student creates an angle of given measure with the given ray as one of its sides (e.g., <math>45^\circ</math>).</p> <p><b>Response Type:</b> Graphing</p>
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<p><b>Task Model 2c</b></p> <p><b>Response Type:</b> Equation/Numeric</p> <p><b>DOK Level 2</b></p> <p><b>4.MD.C.6</b> Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</p> <p><b>Evidence Required:</b> 2. The student uses a protractor to measure angles (composed of one-degree angles) and construct angles to whole-number degrees.</p> <p><b>Tools:</b> None</p>	<p><b>Prompt Features:</b> The student uses a protractor to find the measure of a given angle.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> <li>The orientation of the rays that are presented (e.g., horizontal, pointing to the left and diagonal, pointing to the right)</li> <li>Whether the student is presented with a benchmark (i.e., <math>30^\circ</math>, <math>45^\circ</math>, <math>90^\circ</math>, or <math>180^\circ</math>) or non-benchmark angle</li> </ul> </li> </ul> <p><b>TM2c</b></p> <p><b>Stimulus:</b> The student is presented with an angle imposed on a protractor and given instructions to find the measure of the angle.</p> <p><b>Example Stem:</b> Enter the measure, in degrees, of <math>\angle ABC</math>.</p>  <p><b>Rubric:</b> (1 point) The student enters the correct angle measure, in degrees (e.g., 45).</p> <p><b>Response Type:</b> Equation/Numeric</p>
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<p><b>Task Model 3a</b></p> <p><b>Response Type:</b> Equation/Numeric</p> <p><b>DOK Level 2</b></p> <p><b>4.MD.C.7</b> Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</p> <p><b>Evidence Required:</b> 3. The student decomposes an angle into smaller non-overlapping parts and adds the measures of these smaller parts to find the measure of the whole angle.</p> <p><b>Tools:</b> None</p>	<p><b>Prompt Features:</b> The student enters the measure of a decomposed angle.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>• All angle measures are given and student is asked to find the sum.</li> <li>• Angle measures can be listed or a part of the drawing.</li> <li>• Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> <li>◦ How many non-overlapping parts the angle is decomposed into</li> <li>◦ The orientation of the angle</li> <li>◦ The presence of a horizontal or vertical ray</li> <li>◦ How “friendly” the numbers are to work with</li> </ul> </li> </ul> <p><b>TM3a</b> <b>Stimulus:</b> The student is presented with an angle that is decomposed into non-overlapping parts.</p> <p><b>Example Stem:</b> Use the diagram to solve the problem.</p> <ul style="list-style-type: none"> <li>• The measure of <math>\angle DCE = 50^\circ</math>.</li> <li>• The measure of <math>\angle ECF = 40^\circ</math>.</li> <li>• The measure of <math>\angle FCG = 20^\circ</math>.</li> </ul>  <p>Enter the measure, in degrees, of <math>\angle DCG</math>.</p> <p><b>Rubric:</b> (1 point) The student enters the measure of the whole angle, in degrees (e.g., 110).</p> <p><b>Response Type:</b> Equation/Numeric</p>
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<p><b>Task Model 3b</b></p> <p><b>Response Type:</b> <b>Graphing</b></p> <p><b>DOK Level 2</b></p> <p><b>4.MD.C.7</b> Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</p> <p><b>Evidence Required:</b> 3. The student decomposes an angle into smaller non-overlapping parts and adds the measures of these smaller parts to find the measure of the whole angle.</p> <p><b>Tools:</b> None</p> <p><b>Accessibility Note:</b> Graphing items are not currently able to be Brailled. Minimize the number of items developed to this TM.</p>	<p><b>Prompt Features:</b> The student decomposes an angle.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> <li>How many non-overlapping parts the student decomposes the angle into</li> <li>The orientation of the angle</li> <li>The presence of a horizontal or vertical ray</li> <li>How “friendly” the numbers are to work with</li> </ul> </li> </ul> <p><b>TM3b</b> <b>Stimulus:</b> The student is presented with a protractor showing an angle.</p> <p><b>Example Stem:</b> The protractor shows the measure of <math>\angle PQR</math>. Use the Add Arrow tool to divide <math>\angle PQR</math> into two equal angles.</p>  <p><b>Rubric:</b> (1 point) The student draws a ray that correctly divides the angle into two equal angles (e.g., two <math>60^\circ</math> angles).</p> <p><b>Response Type:</b> Graphing</p>
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<p><b>Task Model 4</b></p> <p><b>Response Type:</b> Equation/Numeric</p> <p><b>DOK Level 2</b></p> <p><b>4.MD.C.7</b> Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</p> <p><b>Evidence Required:</b> 4. The student solves addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.</p> <p><b>Tools:</b> None</p>	<p><b>Prompt Features:</b> The student solves for the unknown angle measure in a problem involving addition or subtraction of angle measures.</p> <p><b>Stimulus Guidelines:</b></p> <ul style="list-style-type: none"> <li>Items that involve using the measure of a straight angle should specify the measure is <math>180^\circ</math>.</li> <li>Item difficulty can be adjusted via these example methods: <ul style="list-style-type: none"> <li>How many non-overlapping parts the student decomposes the angle into</li> <li>The orientation of the angle</li> <li>The presence of a horizontal or vertical ray</li> <li>Whether the item refers to a real-world context or a mathematical context</li> <li>How “friendly” the numbers are to work with</li> </ul> </li> </ul> <p><b>TM4</b> <b>Stimulus:</b> The student is presented with problems in real-world or mathematical contexts involving the use of angle measures of decomposed angles.</p> <p><b>Example Stem 1:</b> Use the figure to answer the question.</p> <ul style="list-style-type: none"> <li>The measure of <math>\angle UWV = 32^\circ</math>.</li> <li><math>\angle TWU</math> is a right angle.</li> <li>The measure of <math>\angle SWV = 180^\circ</math>.</li> </ul>  <p>Enter the measure, in degrees, of <math>\angle SWT</math>.</p> <p><b>Example Stem 2:</b> In the figure shown, <math>JKLM</math> is a rectangle and <math>\angle KJL = 29^\circ</math>.</p>  <p>Enter the measure, in degrees, of <math>\angle MJL</math>.</p> <p><b>Source:</b> Illustrative Mathematics <a href="http://www.illustrativemathematics.org/illustrations/1168">http://www.illustrativemathematics.org/illustrations/1168</a></p>
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**Task Model 4**

**Response Type:**  
Equation/Numeric

**DOK Level 2****4.MD.C.7**

Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.

**Evidence Required:**

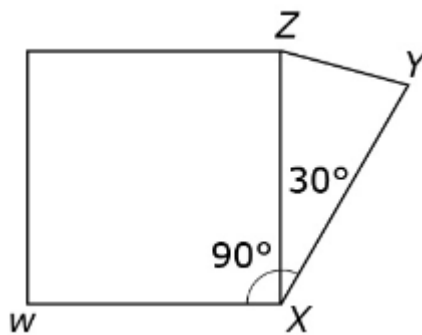
4. The student solves addition and subtraction problems to find unknown angles on a diagram in real-world and mathematical problems.

**Tools:** None

**TM4 (continued)**

**Example Stem 3:** A student made the design shown with shapes.

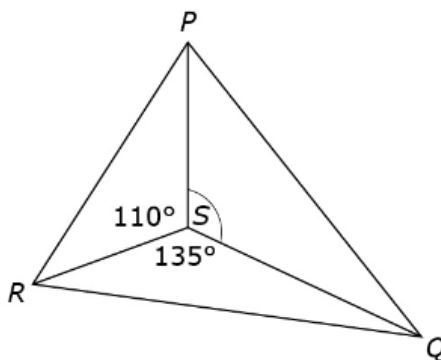
- The measure of  $\angle WXZ = 90^\circ$ .
- The measure of  $\angle YXZ = 30^\circ$ .



Enter the measure, in degrees, of  $\angle WXY$ .

**Example Stem 4:** A student made the design shown with shapes.

- The measure of  $\angle PSR = 110^\circ$ .
- The measure of  $\angle RSQ = 135^\circ$ .



Enter the measure, in degrees, of  $\angle PSQ$ .

**Rubric:** (1 point) The student enters the correct angle measure in degrees (e.g., 58; 61; 120; 115).

**Response Type:** Equation/Numeric