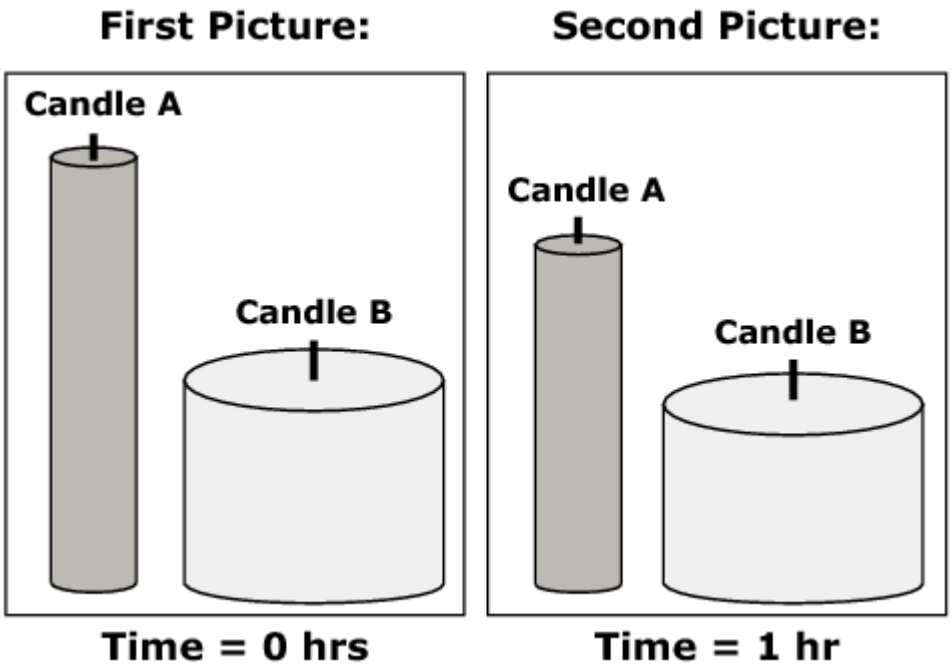
Smarter Balanced Assessment Consortium:

Lights, Candles, Action! Performance Task
Grade 11 Mathematics Practice Test
Scoring Guide
08/14/2019

Lights, Candles, Action!

Your friend Abbie is making a movie. She is filming a fancy dinner scene and she has two types of candles on the table. She wants to determine how long the candles will last.

She takes a picture, lights the candles, and then lets them burn for 1 hour. She then takes a second picture. You can assume that each candle burns at its own constant rate.



Candle Type A initial height = 20 cm
Candle Type B initial height = 10 cm

Candle Type A height after burning for 1 hour = 16 cm
Candle Type B height after burning for 1 hour = 9 cm

You will use this information to help Abbie think about the candles she might use for her film.

1

Candles A and B are lit at the same time. What will be the height, in cm, of each candle after 3 hours of burning?

Candle Type A: ☐

Candle Type B: ☐

2

Candles of each type were lit at the same time. Abbie thinks that since Candle Type A burns more quickly than Candle Type B, that it will burn out (have a height of 0 cm) first.

Julie thinks that since Candle Type B starts out much shorter than Candle Type A, it will be the candle to burn out first.

Which candle will burn out first? Give a mathematical explanation to convince Abbie and Julie of your solution. Clearly identify the quantities involved.

3

Abbie has 3 hours left to film. She lights a **new** Candle Type A and Candle Type B and then starts filming.

In the 3 hours she has left, will Abbie capture the moment when the candles are exactly the same height?

Explain to Abbie how you can determine the answer.

4

You have decided to use functions to help Abbie think about the candles.

You show her how to represent the height of a candle, ***h***, as a function of time, ***t***, using this equation:

$$h = k + nt$$

First, explain to Abbie what **k** and **n** represent in order to model the different candles. Be specific in your explanation.

5

Now, choose either Candle A or Candle B to create an equation that will tell Abbie the height of the candle at ***t*** hours after it is lit.

Determine what the numerical values for **k** and **n** should be for the candle you chose.

Using these **k** and **n** values, write an equation that tells Abbie the height ***h*** of the candle, in cm, at ***t*** hours after it is lit.

6

For her next film, Abbie wants candles that will burn for exactly 8 hours. You want to give her a choice by designing two different candles (Type C and Type D).

Using the equation $h = k + nt$, determine two different pairs of values for **k** and **n** that will meet the requirement to burn down to a height of 0 cm in exactly 8 hours.

Complete the table to show two possible sets of values for **k** and **n** for your new candle designs.

	k	**n**
Candle Type C		
Candle Type D		

1

Candles A and B are lit at the same time. What will be the height, in cm, of each candle after 3 hours of burning?

Candle Type A: ____

Candle Type B: ____

#1 Equation/numeric – 2 response boxes – 1 point for both correct answers

Item	Claim	Domain	Target	DOK	Content	MP	Key
#1	2	RP	D	2	6.RP.A.3	1	Candle A: 8 Candle B: 7

Rubric:
1 point: The student provides *both* correct answers: Candle Type A will be 8 cm and Candle Type B will be 7 cm.

0 points: All other responses

Commentary:
The purpose of the question is to assess whether the student (1) understands the context and the information given and (2) can identify and infer relevant quantities and perform routine calculations.

The context is reasonably authentic. Many students have experience with watching a candle burn and understand the concept of a candle getting shorter as it burns.

Rationale for Content:
This content is securely held, being essentially Grade 6.

> 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Since the question is intended to be accessible for most students, the numbers have been kept simple so as not to create arithmetic computational obstacles in obtaining evidence of Claims 2, 3, or 4.

Rationale for Claim:
The purpose of this first question is to provide an entry-level ramp into the work of the task. This question assesses whether students understand the context, can identify relevant quantities in the given representations, and can perform routine calculations as part of problem solving.

> Claim 2, Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Rationale for DOK:
This is a DOK 2. From the *Depth of Thinking* chart:

> APPLY (DOK 2):
> - Retrieve information to solve a problem
> - Select a procedure and perform it

Because students have to retrieve the information and decide how to use it, this qualifies as DOK 2.

2

Candles of each type were lit at the same time. Abbie thinks that since Candle Type A burns more quickly than Candle Type B, that it will burn out (have a height of 0 cm) first.

Julie thinks that since Candle Type B starts out much shorter than Candle Type A, it will be the candle to burn out first.

Which candle will burn out first? Give a mathematical explanation to convince Abbie and Julie of your solution. Clearly identify the quantities involved.

#2 Short text – 2 points

Item	Claim	Domain	Target	DOK	Content	MP	Key
#2	3	EE	B	3	7.EE.B.4	3	See sample responses

Rubric:
2 points: The student correctly determines that Candle Type A will burn out first AND provides a valid mathematical explanation that includes the initial heights and the burn rates.

Note: The students are not required to calculate the burn out times.

1 point: The student correctly determines that Candle Type A will burn out first, but does not provide a valid mathematical explanation that includes the initial heights and the burn rates.

OR Student correctly reasons from an incorrect calculation.

0 points: All other responses

Commentary:
The purpose of this question is to confront a pair of reasonable sounding arguments. The student is required to use mathematics to justify which of the two claims is correct. The question requires attending to both parameters (initial height and burn rate) of the context and using those to construct a viable argument.

Rationale for Content:
The content is securely held, being Grade 7 Expressions and Equations.

> 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Although students are not specifically asked to solve this problem using an equation, the underlying mathematical structure is that of analyzing linear relationships of the form $px + q = r$.

Rationale for Claim:
The fact that students are given two ideas as to which candle will burn out first and asked to defend which one they agree with based on a mathematical argument is what makes this a Target 3B.

> Claim 3, Target B: Construct, autonomously, chains of reasoning that will justify or refute propositions or conjectures.

Rationale for DOK:
This question qualifies as DOK 3 (Evaluate) because students are required to:

-Cite evidence and develop a logical argument.
-Compare/contrast solution methods.

What follows are sample responses and scoring annotations for Item 2.

Sample Response 2a

Abbie is correct, "because" candle A looks taller, but it burns at a rate of 4cm per hour, where candle B looks shorter and burns at a rate of 1 cm per second. Since candle A is 20 cm tall, it will burn out in 5 hours where candle B would take 10 hours.

SCORE POINT 2

This student correctly identified that Candle Type A would burn out first and used both initial heights and burn rates to compute and compare the burn out times.

Sample Response 2b

Candle A will burn out first becaus candle A burns 4 cms each hour while Candle B burns 1 cm each hour. Candle A will burn out while candle B still has 5cm left.

SCORE POINT 1

Although this student correctly provided the information that 5 cm are left for Candle Type B, he/she did not give any indication as to how that information was derived.

Sample Response 2c

Candle A will burn out faster b/c it get 4 inches shorter every hour when B only get 1 inch shorter.

SCORE POINT **1**

This student correctly stated that Candle Type A would burn out first, but only attended to the burn rates and did not mention the initial heights.

Sample Response 2d

I think candle b will burn out faster because its shorter and fatter candle a is tall in skinny therefore it will take longer to burn out

SCORE POINT **0**

This student only attended to one quantity (height of candle) and concluded that Candle Type B would burn out first, which is incorrect.

3

Abbie has 3 hours left to film. She lights a **new** Candle Type A and Candle Type B and then starts filming.

In the 3 hours she has left, will Abbie capture the moment when the candles are exactly the same height?

Explain to Abbie how you can determine the answer.

#3 Short text – 2 points

Item	Claim	Domain	Target	DOK	Content	MP	Key
#3	2	EE	A	3	8.EE.C.8	2	See sample responses

Rubric:
2 points: The student correctly answers *No*, Abbie will not capture the moment when Candle Type A and Candle Type B are the same height during the 3 hours of filming, AND supports the claim with a mathematically valid argument.

Note: The underlying content is about Systems of Linear Equations; however, students are not required to set up, symbolically, a system of linear equations in order to solve this problem.

1 point: Student correctly answers *No*, Abbie will not capture the moment when Candle Type A and Candle Type B are the same height during the 3 hours of filming, but does not support the claim with a mathematically valid argument.

OR Student correctly reasons from an incorrect calculation.

0 points: All other responses

Commentary:
The purpose of the question is to have students engage with a meaningful context, where the underlying mathematical content is systems of linear equations. By design, the question does not specify or require the student to use any particular solution method. This is to allow more access and opportunities for students to engage in problem solving. By requiring the solution method to be explained, rather than just give an answer, this increases the depth of knowledge.

Rationale for Content:
This content is securely held, being Grade 8 Expressions and Equations.
8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.

Rationale for Claim:
The fact that students are given a context (candles burning) and asked a question (Will the candles reach the same height during a specified time?), the problem is well-posed, but students are not told what technique or procedure they must use to solve.

> Claim 2, Target A: Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace.

Rationale for DOK:
This question qualifies as DOK 3 (Evaluate) because students are asked to:

-Cite evidence and develop a logical argument.
-Compare/contrast solution methods.

Note: If the rubric gave credit for the student merely getting the correct answer, this would qualify as DOK 2, but because the rubric demands a logical argument, this qualifies it as DOK 3.

What follows are sample responses and scoring annotations for Item 3.

Sample Response 3a

1(-4)= -4 2(-4) = -8 3(-4) = -12	1(-1) = -1 2(-1) = -2 3(-1) = -3	No, Abbie will not see the 2 candles at the same height during the 3 hours that she is filming
20-4 = 16 20-8 = 12 20-12 = 8	10-1 = 9 10-2 = 8 10-3 = 7	

SCORE POINT 1

This response is “almost a 2.” In order to move from a score of 1 to a score of 2, this student would need to be more explicit about the meaning of the numbers and calculations, and ultimately connect those to the context.

Sample Response 3b

She will not have enough time to capture both candle having the same height. After 3 hours candle A will be at 8 cm & candle B will be a 6 cm.

SCORE POINT 1

The student correctly states *No*, but does not provide a mathematically valid argument and makes a mistake in the calculation of the height of Candle Type B.

Sample Response 3c

No b/c at 3 hours candel A is 8 cm and B is 7 hours. She would only be able to if she was filming for 4 hours.

SCORE POINT 1

The student correctly states *No,* but does not provide a mathematically valid argument.

Sample Response 3d

I don't know if they will be equal because candle b is burning out faster then candle a. She might be able to capture them burning but not at the same height.

SCORE POINT 0

Student does not state a definitive answer and does not provide a mathematical justification.

4

You have decided to use functions to help Abbie think about the candles.

You show her how to represent the height of a candle, ***h***, as a function of time, ***t***, using this equation:

$$h = k + nt$$

First, explain to Abbie what **k** and **n** represent in order to model the different candles. Be specific in your explanation.

#4 Short text – 2 points

Item	Claim	Domain	Target	DOK	Content	MP	Key
#4	2	F-LE	A, D	2	F-LE.B.5	1	See rubric

Rubric:
2 points: The student correctly identifies that "k" represents the initial height of the candle and "n" represents the burn rate of the candle.

1 point: The student is only able to correctly identify one of the parameters, not both.

0 points: All other responses

Note: It is necessary for the student to identify "k" as the initial or original height in order to receive full credit, because the height of the candle changes as it burns and it is represented by "h" in the equation.

Commentary:
This question is designed to assess a critical aspect of problem solving, namely identifying important quantities in a real-world context and interpreting their meaning in a symbolic representation. For this problem, the students have been provided a linear model and asked to determine the meaning and values of the parameters.

Rationale for Content:
The content is securely held (typically Algebra/Integrated 1), primarily focusing on identifying meaning and finding values of parameters in a linear model. This is part of FLE.B.5:

> Interpret the parameters in a linear or exponential function in terms of a context.

Rationale for Claim:
The purpose of this question is to assess whether students can determine the meaning of parameters in a linear function. Because the form of the linear function is explicitly given to the students, this question fits the criteria for Claim 2, Targets A and D.

> Claim 2, Target A: Apply mathematics to solve well-posed problems in pure mathematics and arising in everyday life, society, and the workplace.
>
> Claim 2, Target D: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Rationale for DOK:
This question qualifies for a DOK 2, Understand, because students are asked to specify and explain relationships.

5

Now, choose either Candle A or Candle B to create an equation that will tell Abbie the height of the candle at ***t*** hours after it is lit.

Determine what the numerical values for **k** and **n** should be for the candle you chose.

Using these **k** and **n** values, write an equation that tells Abbie the height ***h*** of the candle, in cm, at ***t*** hours after it is lit.

#5 Equation/numeric – 1 point

Item	Claim	Domain	Target	DOK	Content	MP	Key
#5	4	F	F	2	8.F.B	4	Either $h = 20 - 4t$ or $h = 10 - t$ (and equivalents)

Rubric:
1 point: The student provides either $h = 20 - 4t$ or $h = 10 - t$ or equivalent.

0 points: All other responses

Commentary:
Students need to understand that the candles are characterized by two parameters: initial height and burn rate. Students must identify a correct initial height and burn rate. This question is designed for students to arrive at a single answer. Students will need to understand how the height, as a function of time, depends on these two parameters. Students will need to interpret the burn time algebraically in order to create an equation, which labels this item as mathematical modeling as opposed to problem solving.

Rationale for Content:
The content is securely held from Grade 8, focusing on identifying meaning and finding values of the parameters in a linear model. This is part of 8.F.B:
Use functions to model relationships between quantities.

Rationale for Claim:
This question aligns with Claim 4, Target F.
Claim 4, Target F: Identify important quantities in a practical situation and map their relationships (e.g., using diagrams, two-way tables, graphs, flowcharts, or formulas).

Rationale for DOK:
Since the student needs to determine an equation that models the height of the candle, this aligns to DOK 2.

DOK 2: Understand, because students are asked to specify and identify a relationship.

6

For her next film, Abbie wants candles that will burn for exactly 8 hours. You want to give her a choice by designing two different candles (Type C and Type D).

Using the equation $h = k + nt$, determine two different pairs of values for k and n that will meet the requirement to burn down to a height of 0 cm in exactly 8 hours.

Complete the table to show two possible sets of values for **k** and **n** for your new candle designs.

	k	**n**
Candle Type C		
Candle Type D		

#6 Fill-in table – 2 points

Item	Claim	Domain	Target	DOK	Content	MP	Key
#6	4	F-LE	E	3	F-LE.5.B	4	Many sets of values work.

Rubric:
2 points: The student creates values for k and n that result in Candle Type C *and* Candle Type D burning out in 8 hours.

1 point: The student creates values for k and n that result in Candle Type C *or* Candle Type D burning out in 8 hours.

0 points: All other responses

Commentary:
As the culminating question in this task, the students need to understand that candles are characterized by two parameters: initial height and burn rate. This question is designed to have infinitely many solutions; hence it is not the type of problem where students solve an equation and arrive at a single answer. Students will need to understand how the height, as a function of time, depends on these two parameters. Students will have to interpret the constraint (burn time of 8 hours) algebraically in order to create examples that satisfy this constraint. It is this aspect of the work that pushes this question into mathematical modeling as opposed to problem solving.

Note that this problem provides some scaffolding by asking students to fill in a table with the parameters. Based on piloting, this was to help students focus on the mathematical work of finding parameters that met the requirement, rather than spend time trying to decide what the parameters were or what specifies a candle.

Also, even though this is a high school task, the numbers have been kept purposefully simple in order to have the students focus on the design under constraint element of this work, rather than the arithmetic complexity.

Rationale for Content:
The content is securely held (typically Algebra/Integrated 1), primarily focusing on identifying meaning and finding values of parameters in a linear model. This is part of FLE.B.5:

> Interpret the parameters in a linear or exponential function in terms of a context.

Rationale for Claim:
This question aligns with Claim 4, Target E because this problem asks students to develop a model of a real phenomenon (at least parts of the model). This particular question is asking students to design candles by specifying values of essential parameters in a linear model.

> Claim 4, Target E: Analyze the adequacy of and make improvements to an existing model or develop a mathematical model of a real phenomenon.

Rationale for DOK:
Since the student needs to design new candles while meeting certain requirements, finding parameters to meet given constraints in a context is asking students to apply and create. For these reasons, this question would satisfy the requirements for DOK 3.

DOK 3:
Apply: Use reasoning, planning, and supporting evidence
Create: Develop an alternative solution

Sample full-credit responses:

	k	**n**
Candle Type C	16	-2
Candle Type D	8	-1

	k	**n**
Candle Type C	12	-1.5
Candle Type D	24	-3

The hand-scored items in this guide are 2-point short-text items. The general rubric that is used as a basis for scoring all 2-point short-text items is shown below. Although item-specific rubrics are also provided to scorers to facilitate the hand-scoring of short-text items, every response should be able to map back to this general rubric in a consistent and reliable manner.

Smarter Balanced Mathematics General Rubric for 2-Point Items

Score	Description
2	The student has demonstrated a **full and complete** understanding of all mathematical content and practices essential to this task. The student has addressed the task in a mathematically sound manner. The response contains evidence of the student's competence in problem solving, reasoning, and/or modeling to the full extent that these processes apply to the specified task. The response may, however, contain minor flaws that do not detract from a demonstration of full understanding.
1	The student has demonstrated a **partial** understanding of the mathematical content and practices essential to this task. The student's response contains some of the attributes of an appropriate response but lacks convincing evidence that the student fully comprehends the essential mathematical ideas addressed by this task. Such deficits include evidence of insufficient mathematical knowledge; errors in fundamental mathematical procedures; and other omissions or irregularities that bring into question the student's competence in problem solving, reasoning, and/or modeling related to the specified task.
0	The student has demonstrated **merely an acquaintance** with the topic, or provided a completely incorrect or uninterpretable response. The student's response may be associated with the task, but contains few attributes of an appropriate response. There are significant omissions or irregularities that indicate a lack of comprehension in regard to the mathematical content and practices essential to this task. No evidence is present that demonstrates the student's competence in problem solving, reasoning, and/or modeling related to the specified task.